Section 3.4

The Product Rule

Consider the flight of a (unrealistic) two-stage model rocket and let $t = 0$ represent the moment the first stage is released. The rocket burns fuel for 12 sec, decreasing the overall mass at a constant rate of 0.2 kg/s from an initial 3 kg. The square of the velocity increases at a constant rate of 5 m/s with initial velocity $v(0) = 2$ m/s. Find a formula for the momentum P as a function of time.

The derivative of $P(t)$ at time t is the instantaneous rate of change in momentum at the given time. How might $v'(t)$ and $m'(t)$ affect or contribute to $P'(t)$?

Write $P'(t)$ if $m(t)$ was constant; if $v(t)$ was constant.

We can visualize P as the area of a rectangle of side lengths m and v. Click here to view this rectangle in Desmos and explore $P'(t)$.

Theorem 3.4.1 (The Product Rule)**.** If f and g are differentiable at x, then

$$\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x).$$

Example 3.4.2. Use the product rule to compute the derivatives of the following functions:

1. $P(t) = (3 - .2t)(\sqrt{5t + 4})$

2. $f(x) = x(x - 1)$

3. $g(x) = 6x - 2xe^x$

The Quotient Rule

Consider $Q(x) = \dfrac{f(x)}{g(x)}$. How can we find $Q'(x)$?

Theorem 3.4.3 (The Quotient Rule)**.** If f and g are differentiable at x and $g(x) \neq 0$, then

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - g'(x)f(x)}{(g(x))^2}$$

Example 3.4.4. Use the quotient rule to compute the derivatives for the following functions:

1. $f(x) = \dfrac{x^2 + 3x + 4}{x^2 - 1}$

2. $g(x) = \dfrac{e^x}{e^x + 1}$

3. $h(x) = xe^{-x}$

4. $Q(x) = \dfrac{4xe^x}{x^2 + 1}$